Polarization

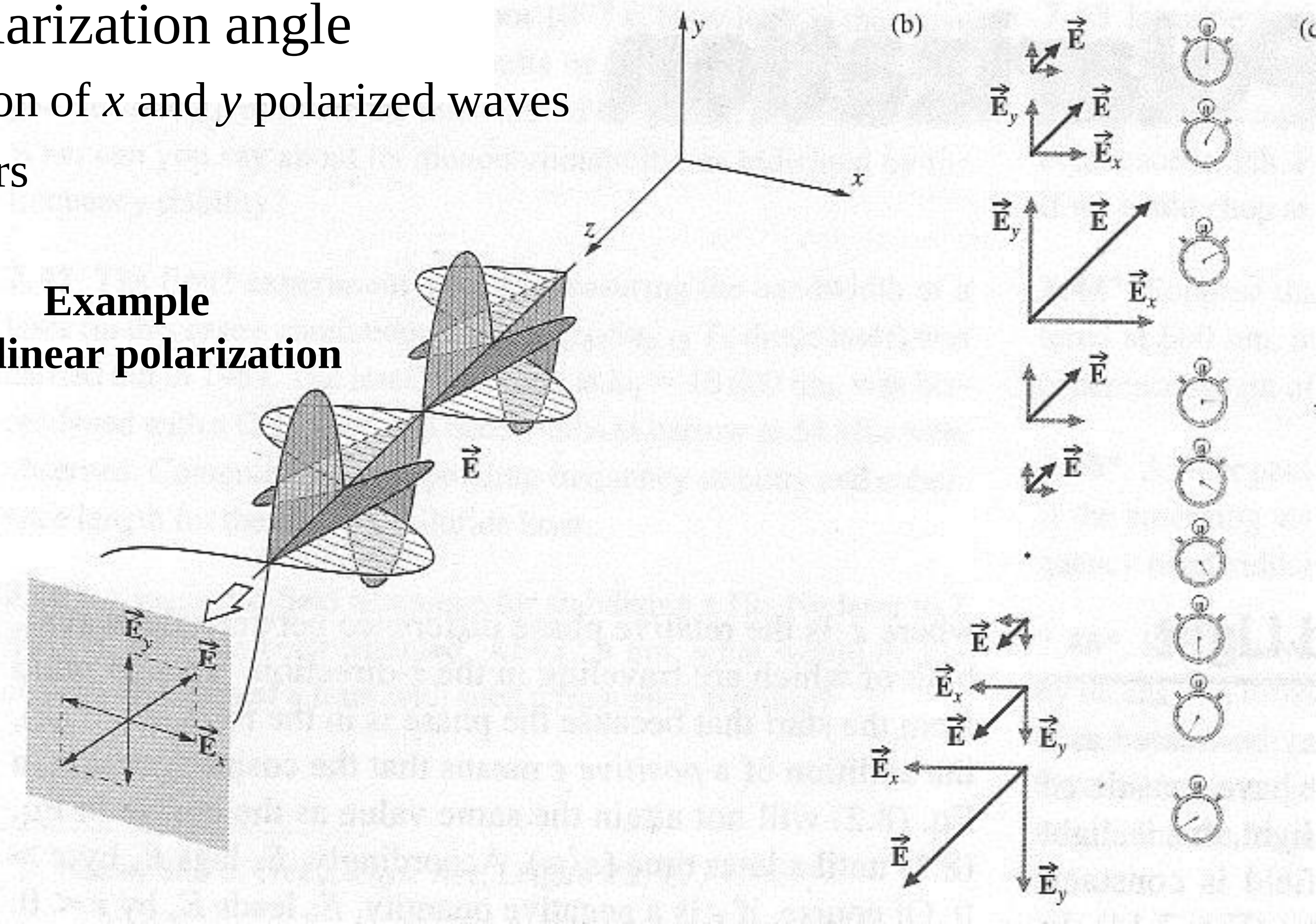
Optics, Eugene Hecht, Chpt. 8

Linear polarization

- E-field magnitude oscillates
- Direction fixed
- Arbitrary polarization angle
  - superposition of x and y polarized waves
  - real numbers

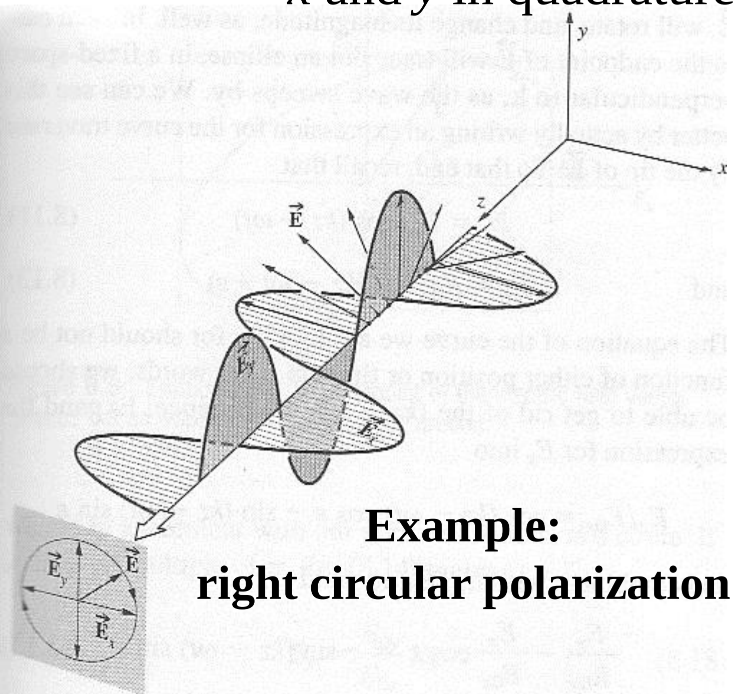
Time evolution

Example
45 ° linear polarization



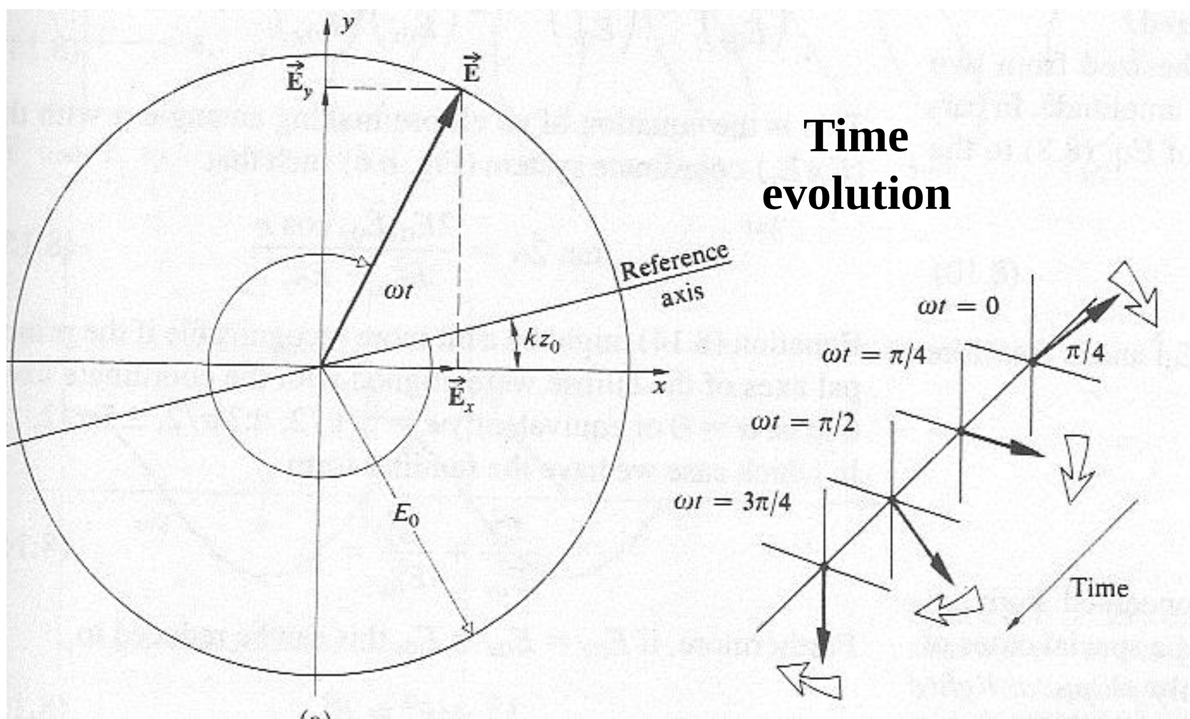
Circular polarization

- E-field magnitude constant
- Direction rotates
- Complex superposition of *x* and *y* polarizations
  - *x* and *y* in quadrature



Example:
right circular polarization

Figure 8.3 Right-circular light. (*a*) Here the electric field, which has a constant amplitude, rotates clockwise with the same frequency with which it oscillates. (*b*) Two perpendicular antennas radiating with a 90° phase difference produce circularly polarized electromagnetic waves.

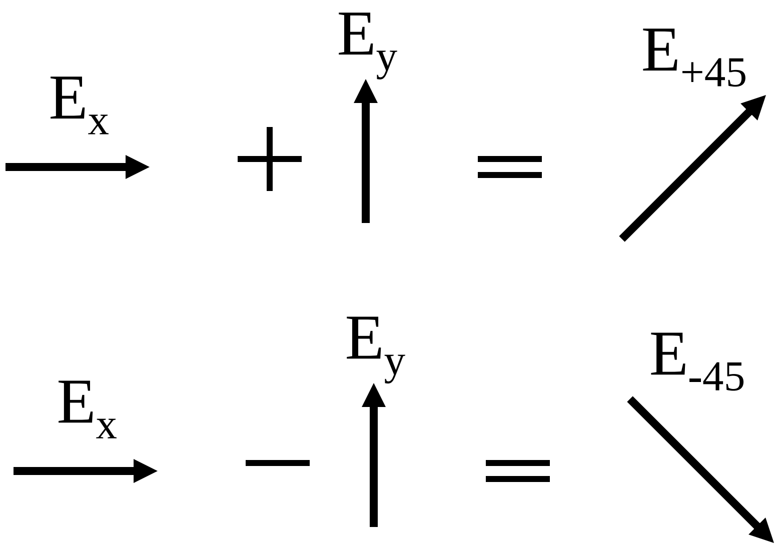


Time evolution

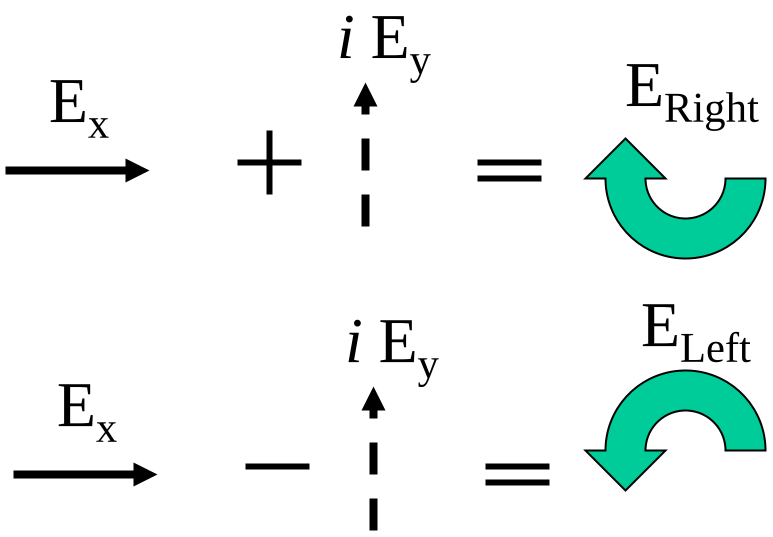
Polarization summary

- Decompose into *x* and *y* polarizations
- Linear -- real superposition
- Circular -- quadrature superposition

Linear polarizations



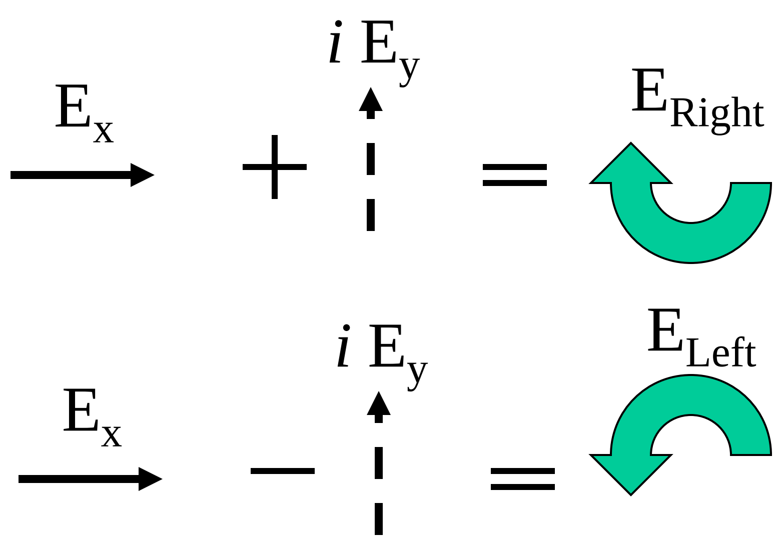
Circular polarizations



Angular momentum of light -- Spin

- Circular polarized light has angular momentum
- Like spin
- Can induce electron spin flips
  - important for spectroscopy etc.

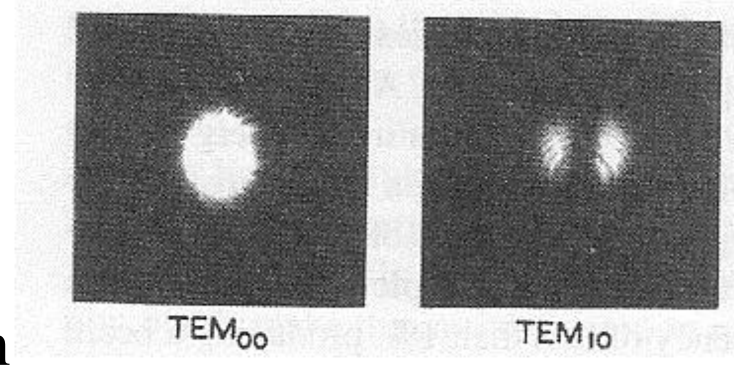
Circular polarizations



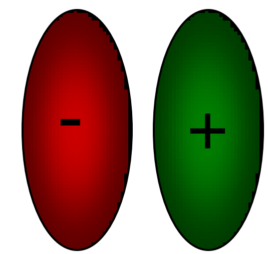
Light angular momentum

- Transverse laser modes
- Addition rules
  - similar to polarization
- Quadrature case
  - wavefront is helix
- "Orbital" angular momentum

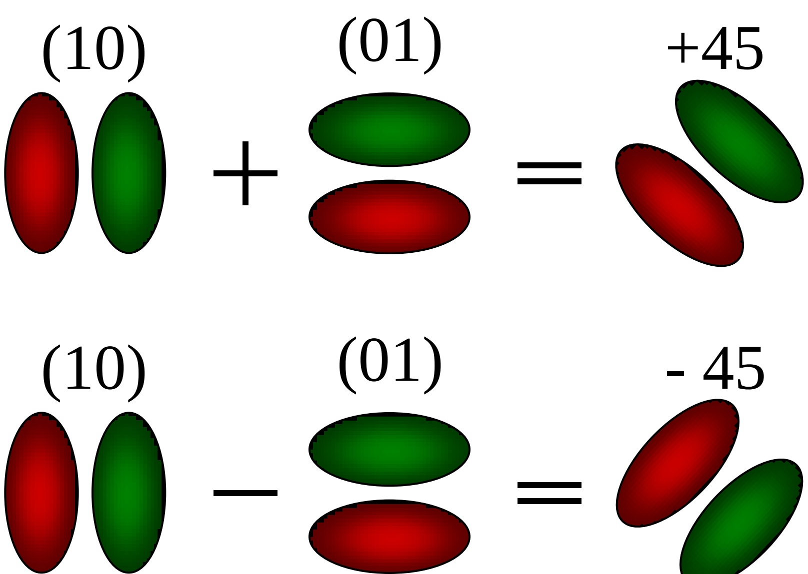
Transverse laser modes



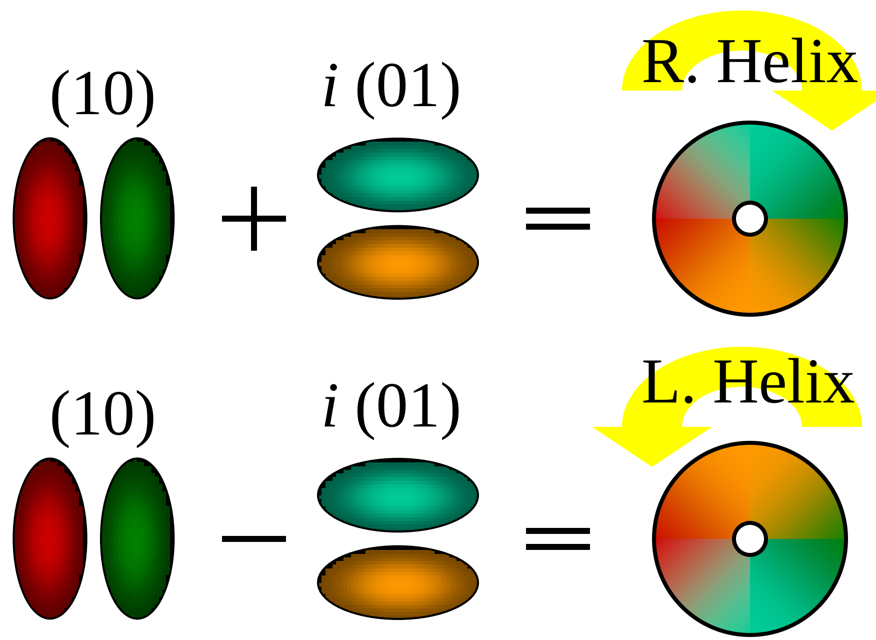
(10) mode



Linear addition

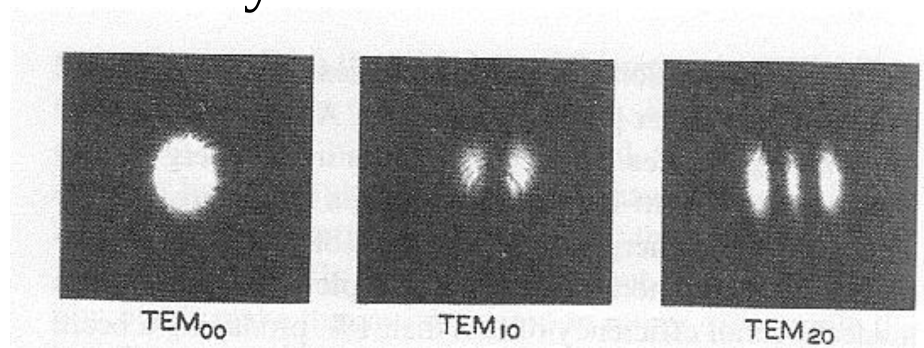


Quadrature addition

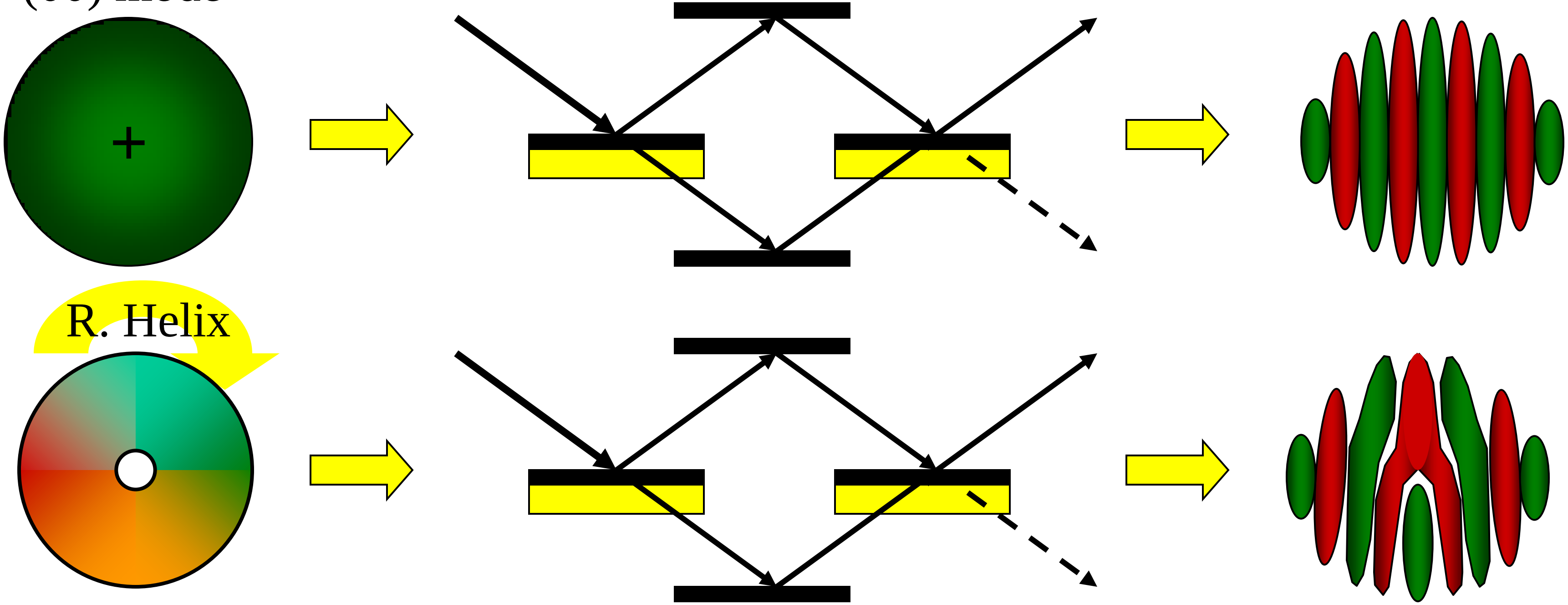


Properties of helical beams

- Interference fringes have discontinuity
- Can have higher order
- Number of extra fringes
  - order number



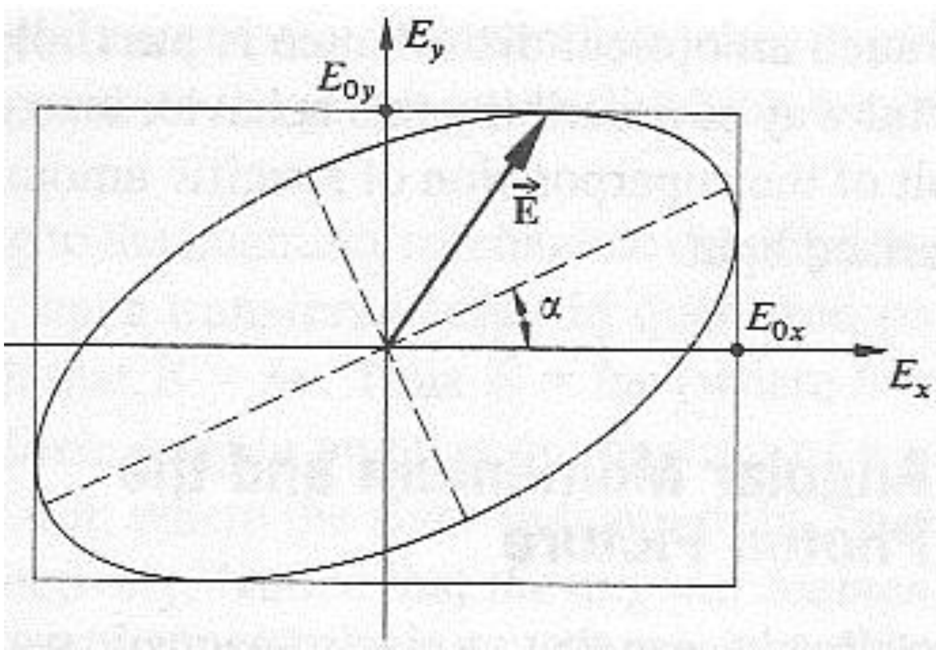
(00) mode



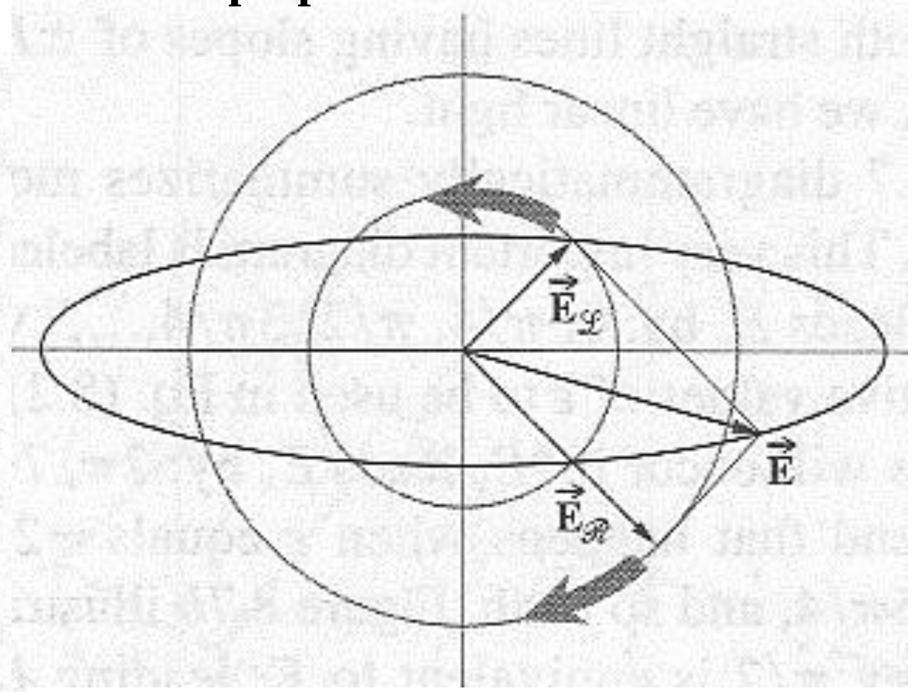
Elliptical polarization

- General case
  - polarization partly linear and partly circular
- E-field sweeps out ellipse
  - both magnitude and direction change with time
- Superposition of *L* and *R* states

Elliptical polarization



Superposition of *L* and *R*



Producing linear polarization -- 1

- Induce loss for one polarization direction
- Examples: wire grid, polaroid filter

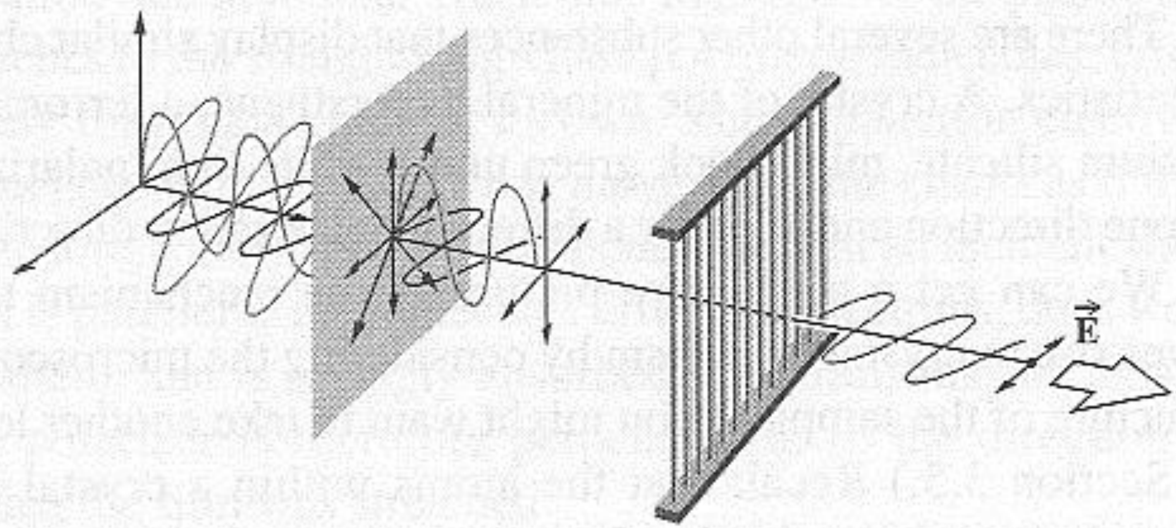


Figure 8.12 A wire-grid polarizer. The grid eliminates the vertical component (i.e., the one parallel to the wires) of the *E*-field and passes the horizontal component.

Producing linear polarization -- 2

- Separation in birefringent crystal -- ex: calcite
- Ordinary wave -- behaves as expected
- Extra-ordinary wave -- behaves differently
  - example -- E-field not perpendicular to propagation direction
- Input light converted to two polarized beams

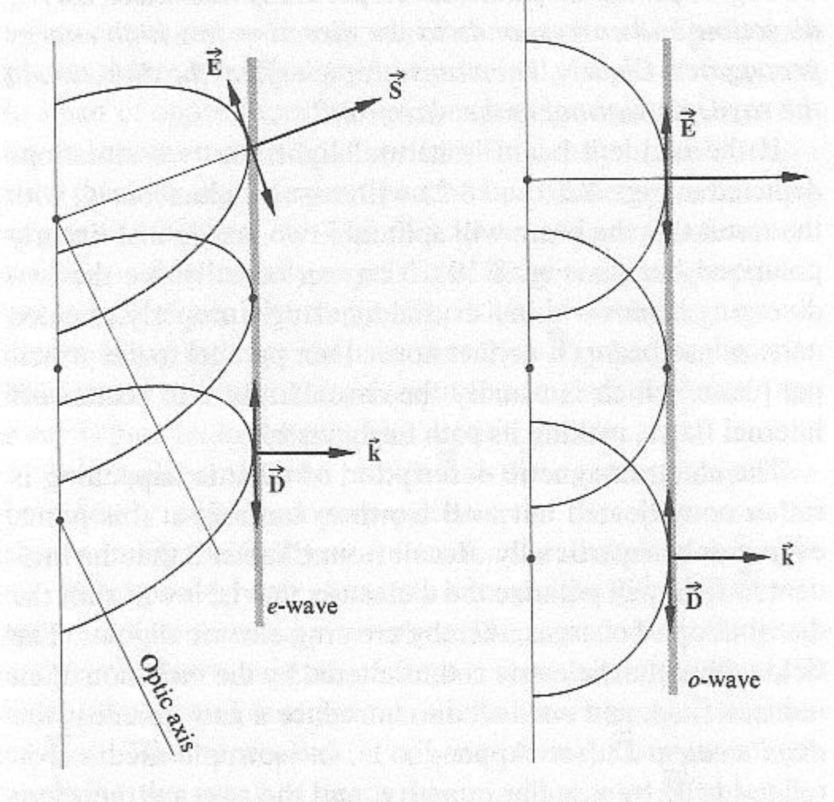
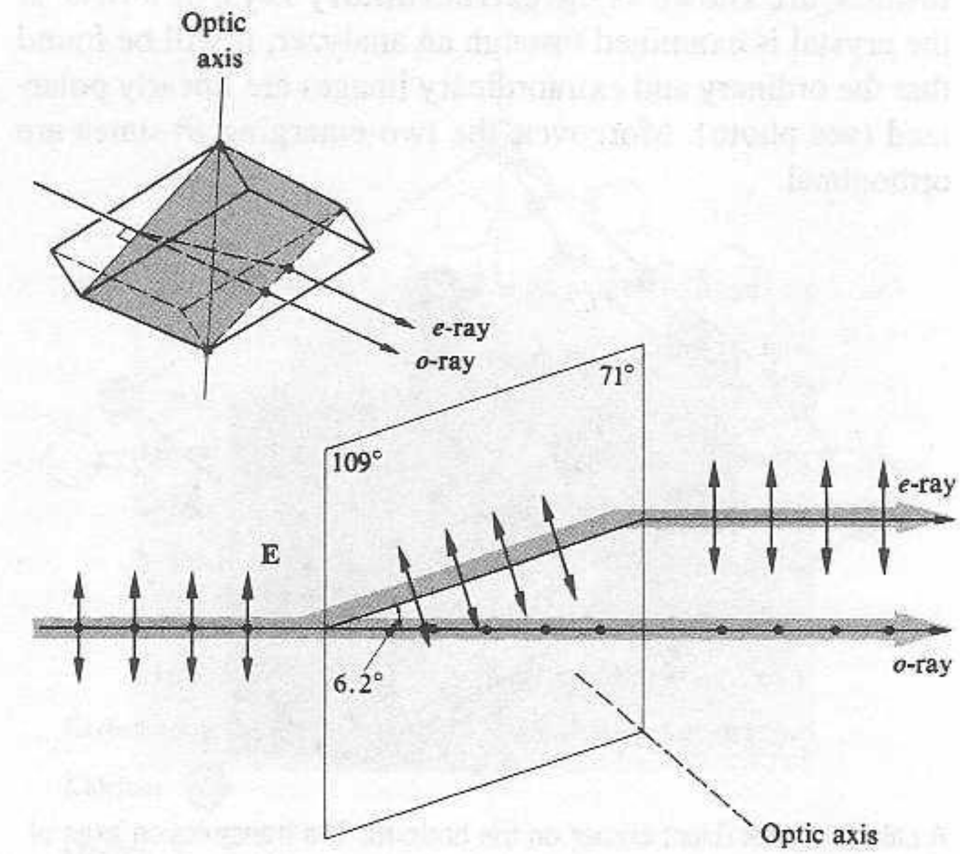


Figure 8.23 Orientations of the $\vec{E}$-, $\vec{D}$-, $\vec{S}$-, and $\vec{k}$-vectors.



Producing linear polarization -- 3

- Brewster's angle
- Only one polarization reflected
- Reflected light polarized
- Works with most surfaces
- Good way to calibrate polarizers

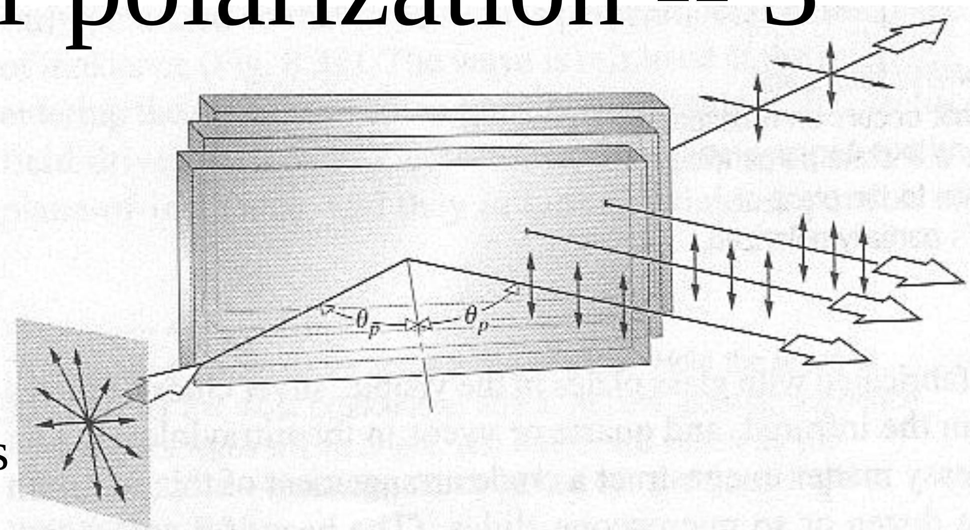
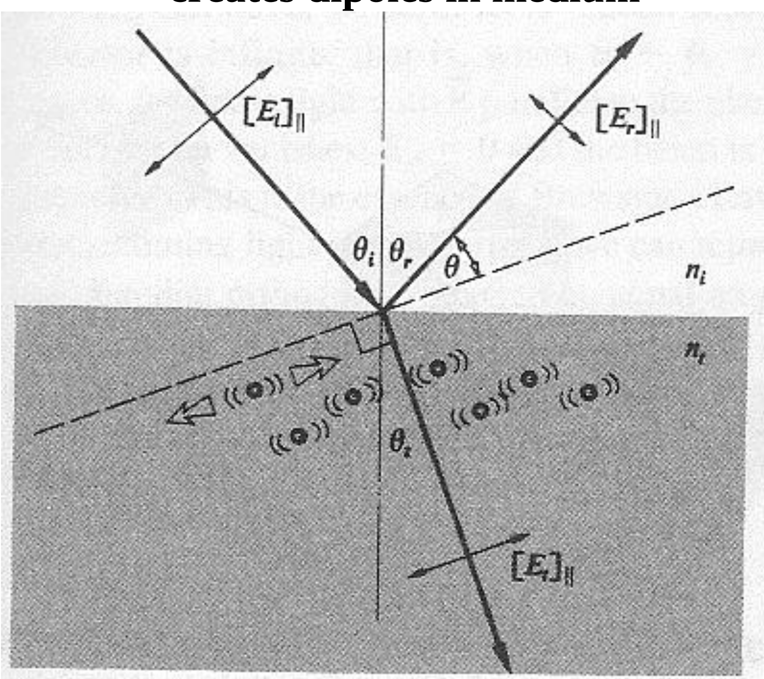
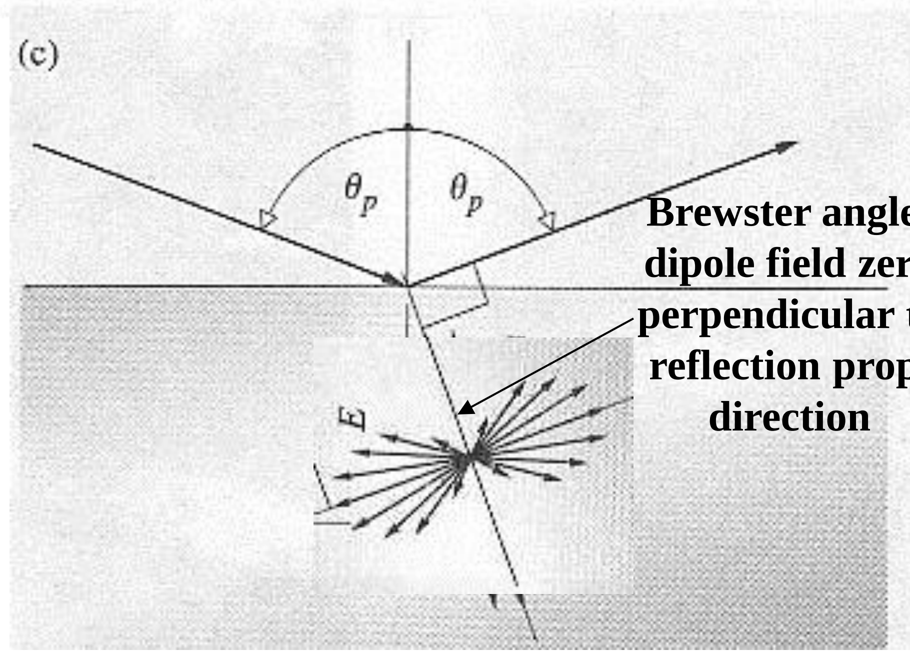


Figure 8.33 The pile-of-plates polarizer.

Refracted beam creates dipoles in medium





Brewster angle: dipole field zero perpendicular to reflection prop. direction

Polarizing cubes

- Uses fact that total internal reflection close to Brewster's angle
  - reflection large for other polarization
- Multi-layer coating enhances effect
  - reflections from multiple surfaces -- resonance
- Brewster polarization reflection always zero

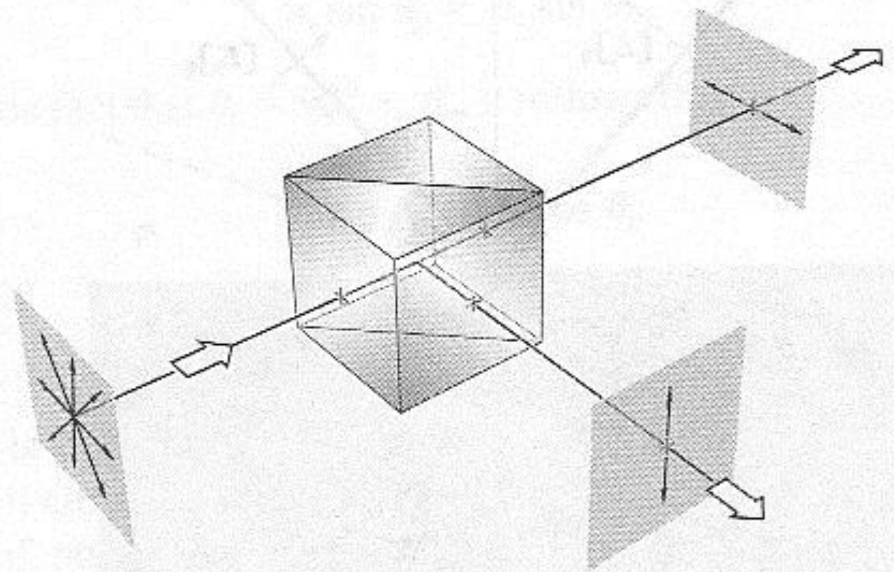


Figure 8.34 A polarizing cube contains a mutilayer dielectric thin film structure on its diagonal face. Reflection from that structure polarizes the incident light, much as would a pile-of-plates.

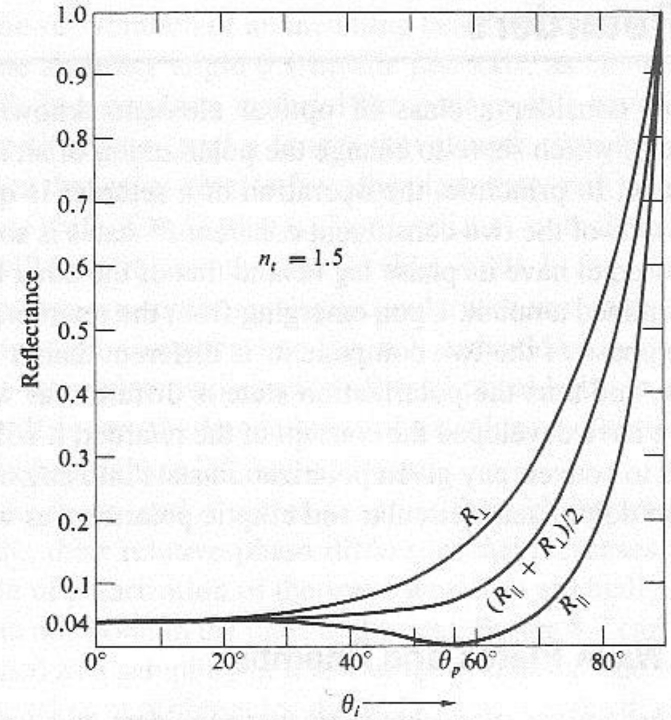
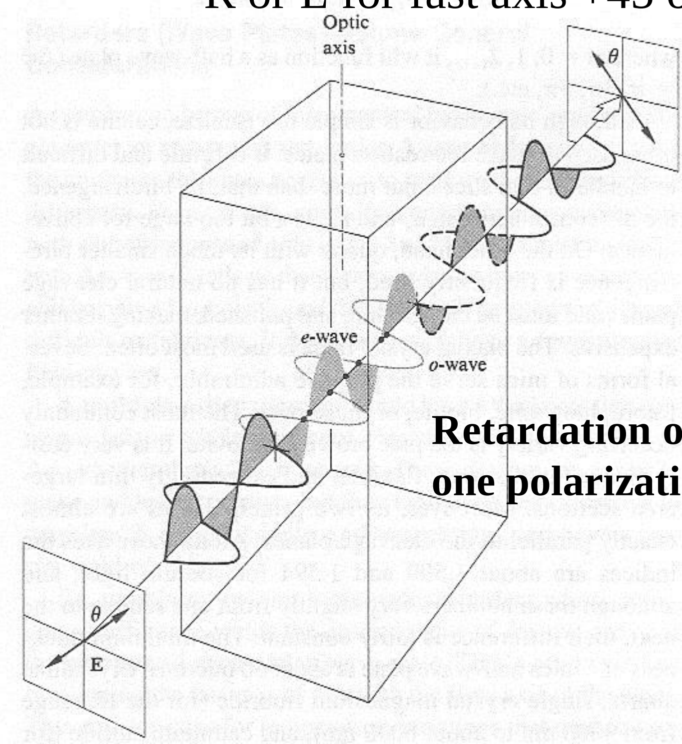


Figure 8.35 Reflectance versus incident angle.

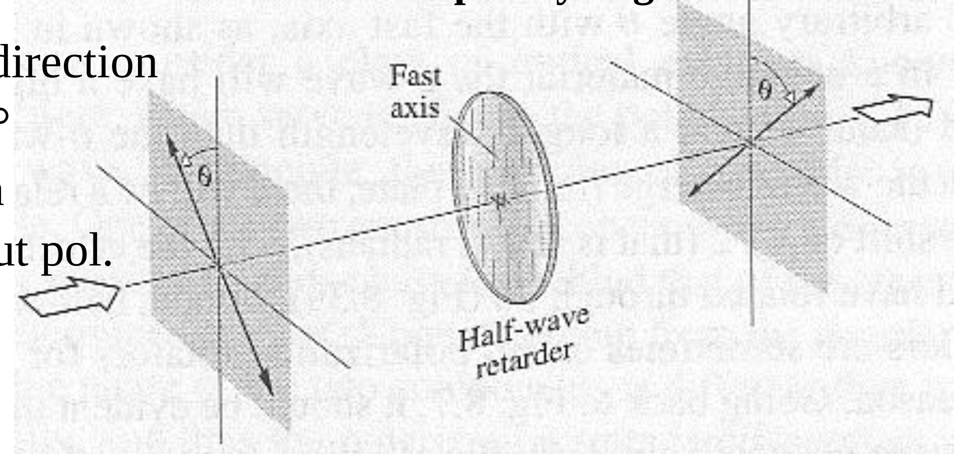
Waveplates

- Polarization converters
- One linear polarization direction propagates faster
- Half wave plate -- phase delay 180°
  - rotate linear polarization up to 90°
  - fast axis at 45° to input polarization direction
- Quarter wave plate -- phase delay 90°
  - convert linear to circular polarization
  - R or L for fast axis +45 or -45 to input pol.

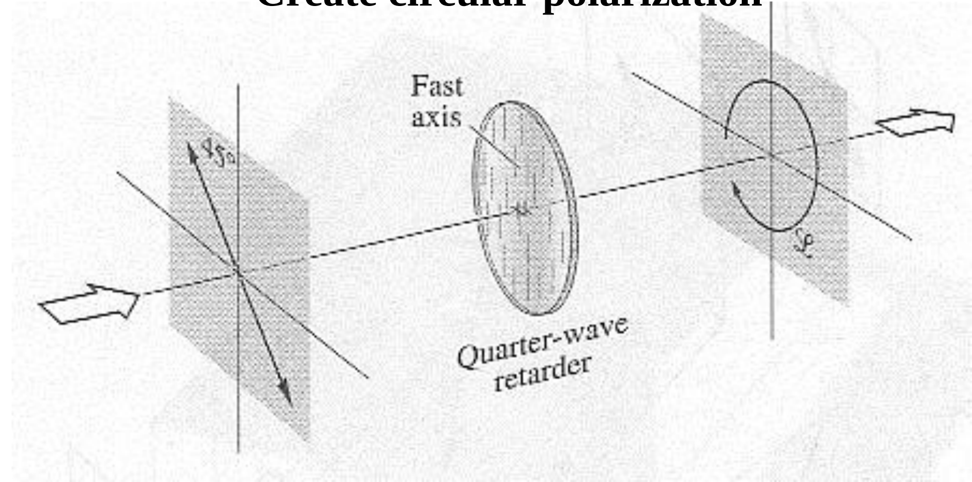


Retardation of one polarization

Rotate linear pol. by angle 2θ



Create circular polarization



Other circular polarizers

- Use phase shift for total internal reflection
  - 45° over broad range of angles
- Two reflections give 90°
- Converts linear to circular polarization

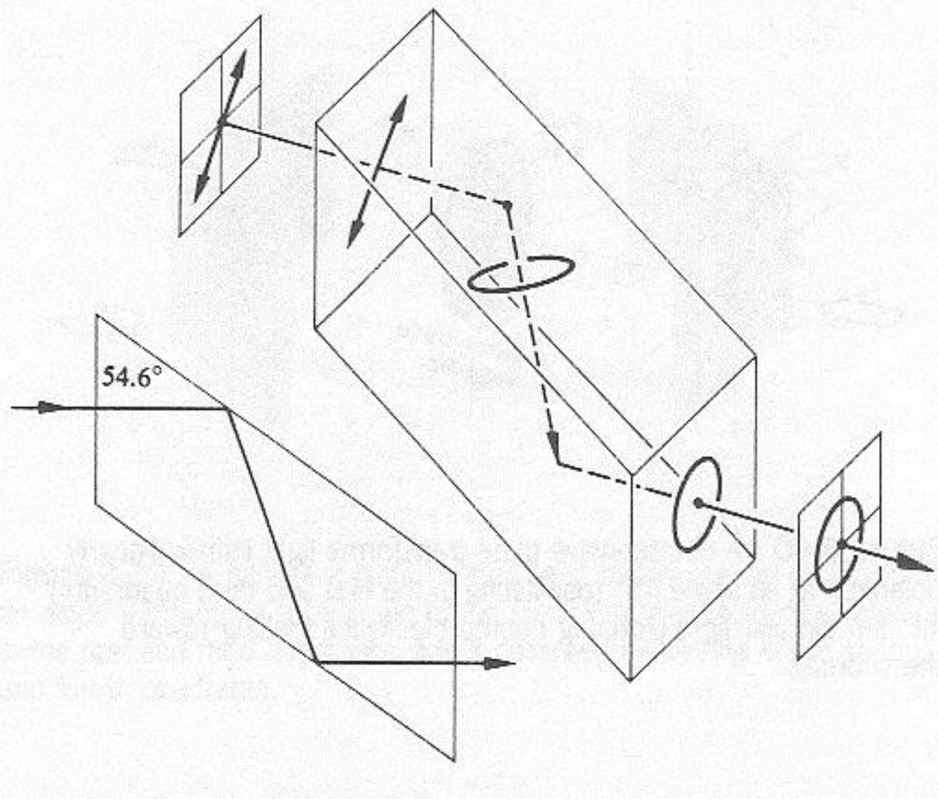


Figure 8.41 The Fresnel rhomb.

Optical activity

- Rotate linear polarization
- Express linear as sum of R and L
- Different propagation speeds
- Phase delays give rotation

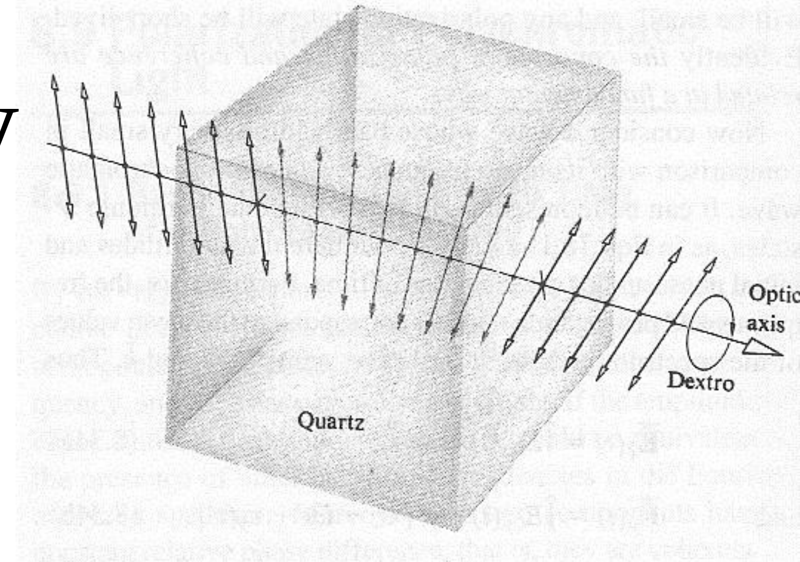
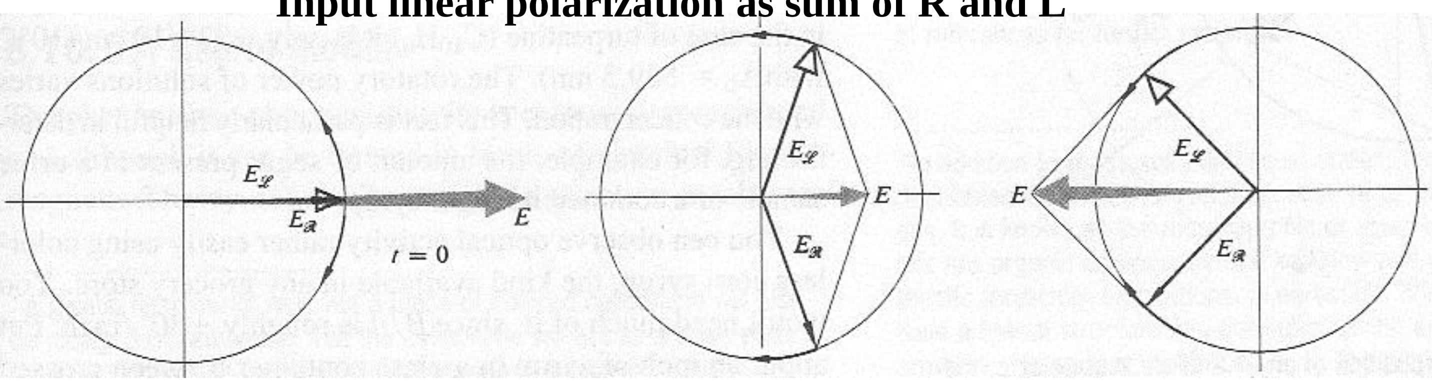
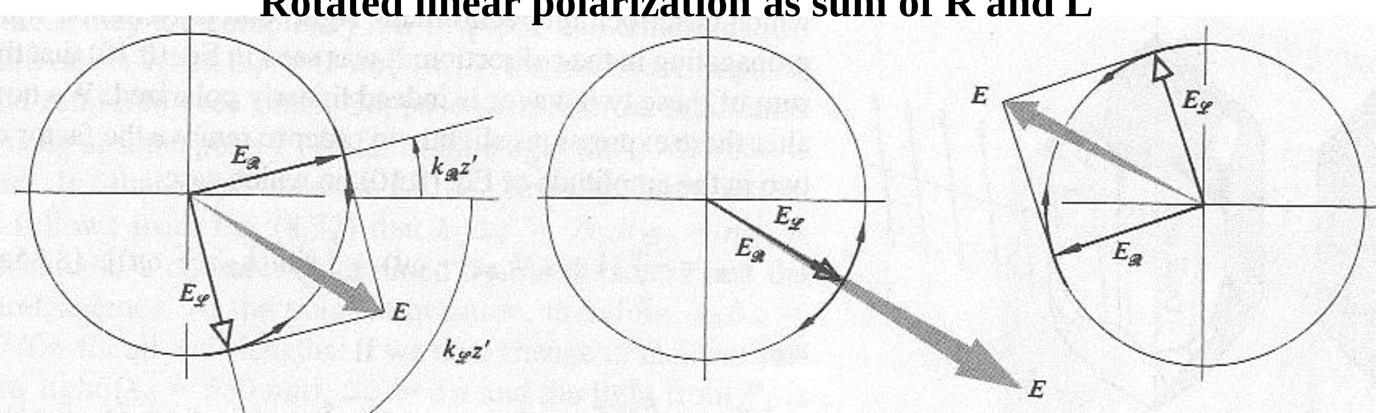


Figure 8.47 Optical activity displayed by quartz.

Input linear polarization as sum of R and L



Rotated linear polarization as sum of R and L



Uses of optical activity

- Organic molecule ID
  - right and left handed molecules
  - Example: helical molecule
- Biological molecule ID
  - almost always pure right or left
  - not mixture

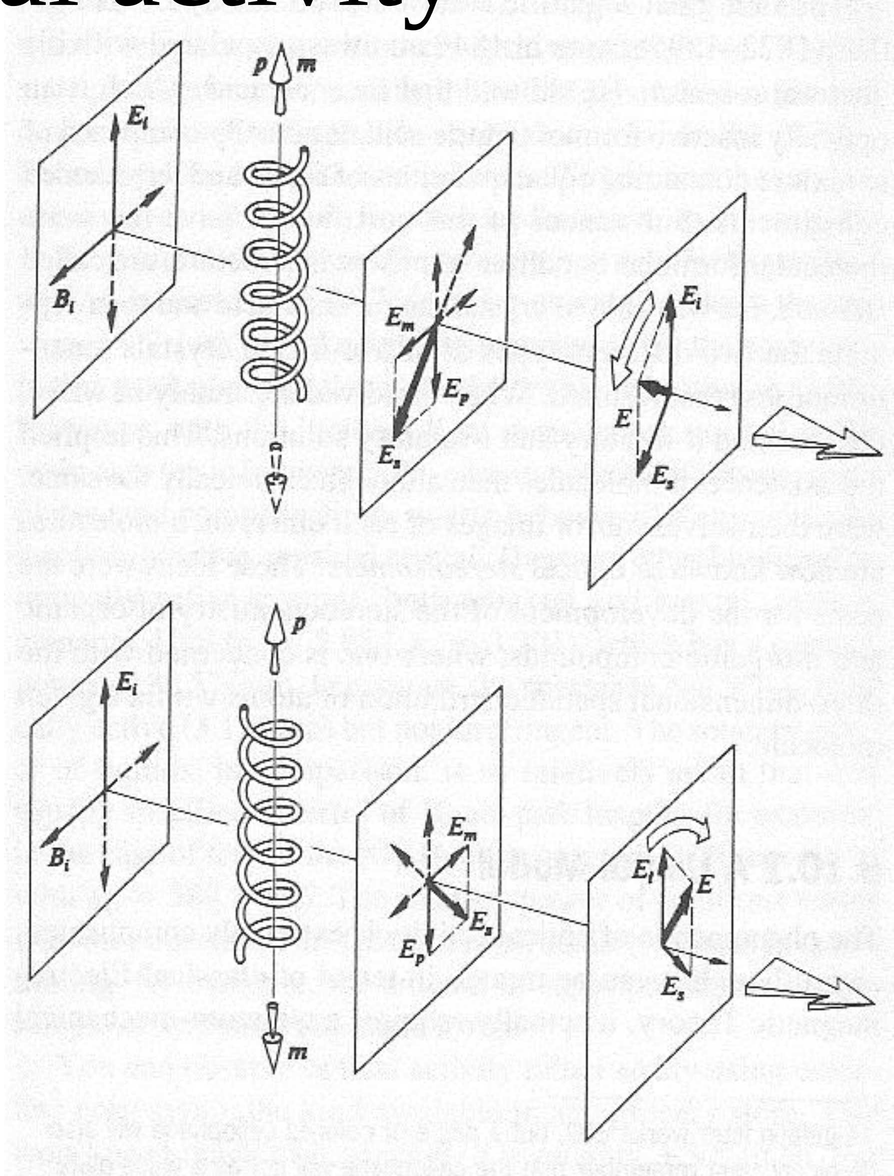
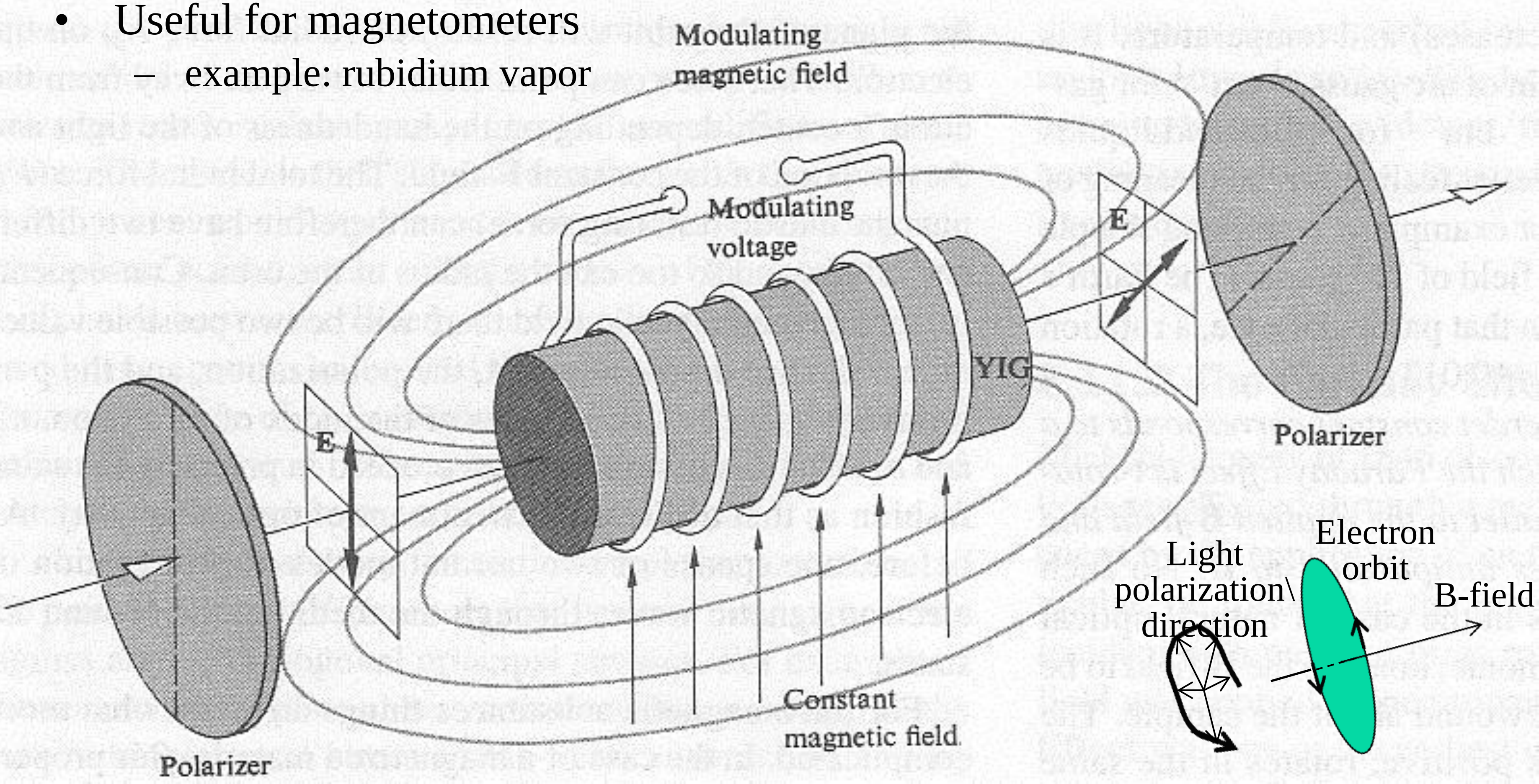


Figure 8.54 The radiation from helical molecules.

Faraday effect

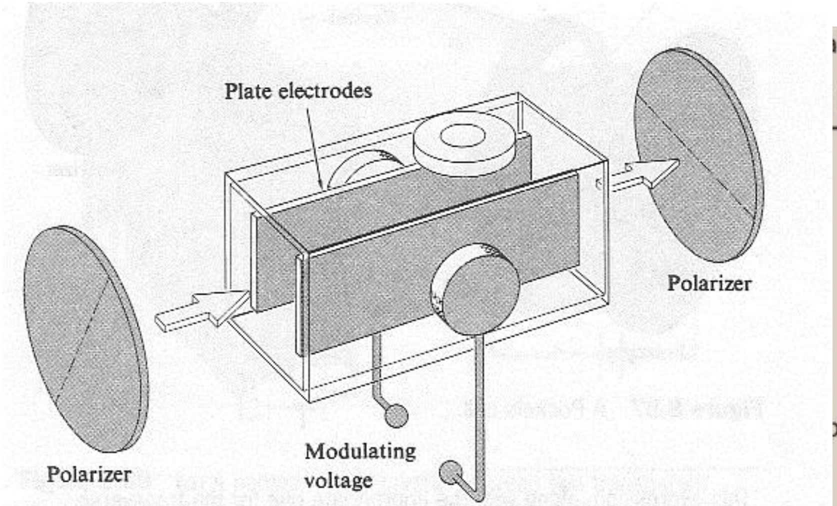
- Magnetic field induces polarization rotation
- Orients electron spins in medium
- Angular momenta of electrons and photons interact
- R and L have different propagation delays
- Useful for magnetometers
  - example: rubidium vapor

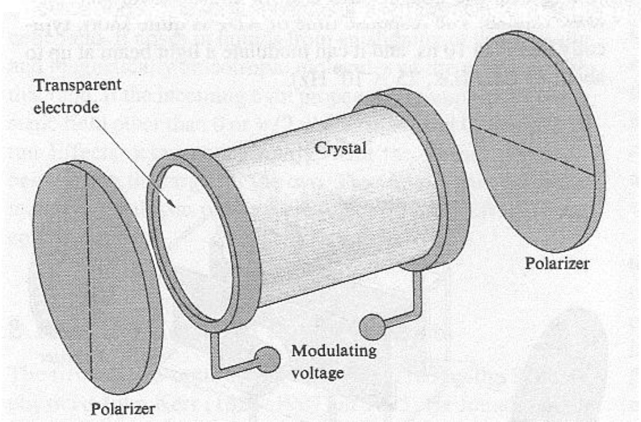


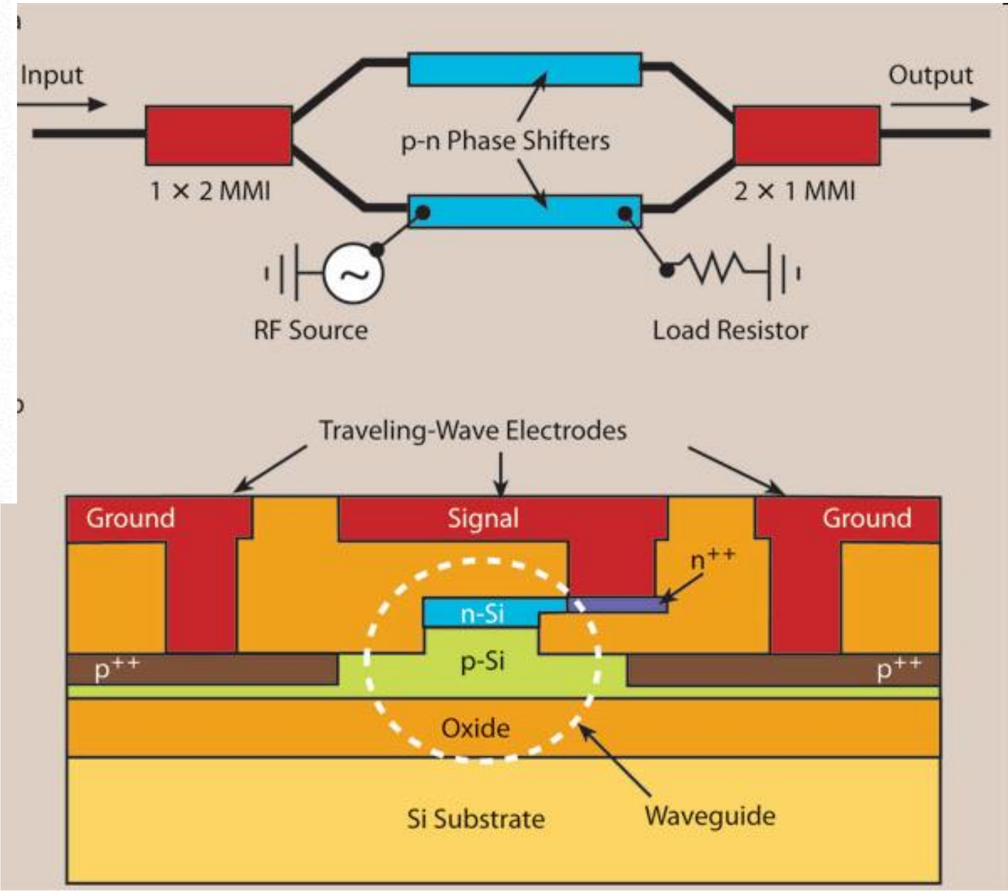
Pockels effect

Electro optic effect – linear in applied field

- Polarization in direction of applied field changes propagation speed
- Requires crystal with no center of symmetry -- also piezoelectric
- Delay linear in applied field -- Kerr effect quadratic







Kerr effect

Retardation nonlinear in applied voltage

All materials can have Kerr effect, but need higher voltage

- In direction of applied field -- phase modulator
- Perpendicular to applied field -- nothing
- 45° to applied field -- variable waveplate
  - output polarizer gives intensity modulator

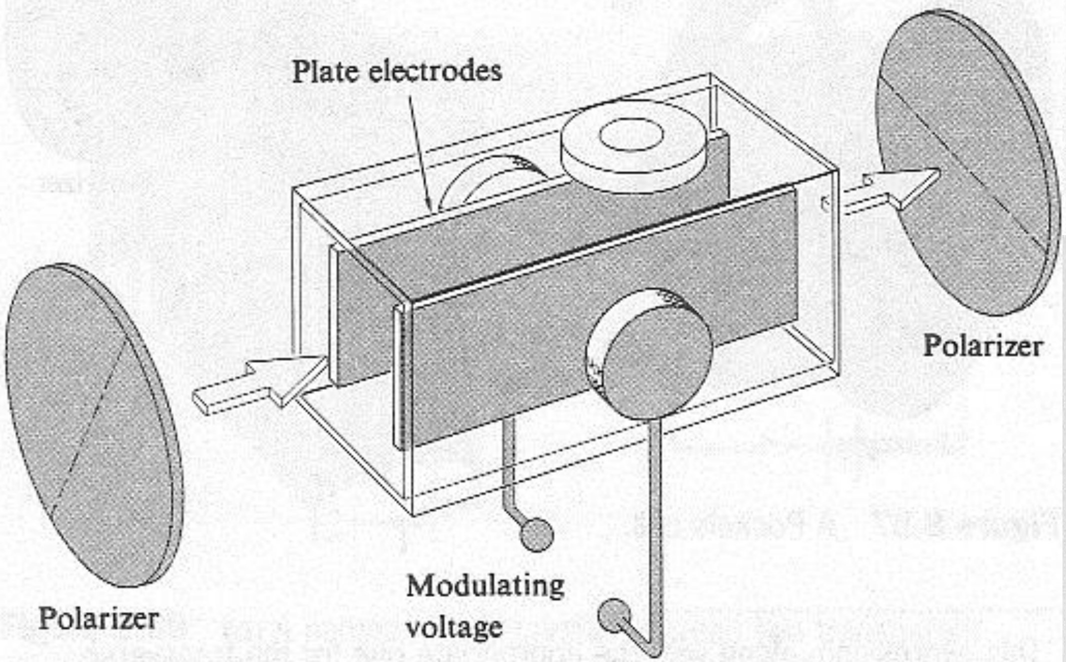


TABLE 8.4 Electro-optic Constants (Room Temperature, $\lambda_0 = 546.1$ nm)

Material	r_{63} (units of 10^{-12} m/V)	n_o (approx.)	$V_{\lambda/2}$ (in kV)
ADP ($NH_4H_2PO_4$)	8.5	1.52	9.2
KDP (KH_2PO_4)	10.6	1.51	7.6
KDA (KH_2AsO_4)	~13.0	1.57	~6.2
KD*P (KD_2PO_4)	~23.3	1.52	~3.4

TABLE 8.3 Kerr Constants for Some Selected Liquids (20°C, $\lambda_0 = 589.3$ nm)

	Substance	K (in units of 10^{-7} cm statvolt^{-2})
Benzene	C_6H_6	0.6
Carbon disulfide	CS_2	3.2
Chloroform	$CHCl_3$	−3.5
Water	H_2O	4.7
Nitrotoluene	$C_5H_7NO_2$	123
Nitrobenzene	$C_6H_5NO_2$	220

Liquid crystals

- Electric field changes average orientation of molecules
  - Delay depends on polarization direction
  - Kerr effect most common
- Phase modulator or variable waveplate
- Intensity modulator needs polarizers
- Used for displays -- ex: computer monitors

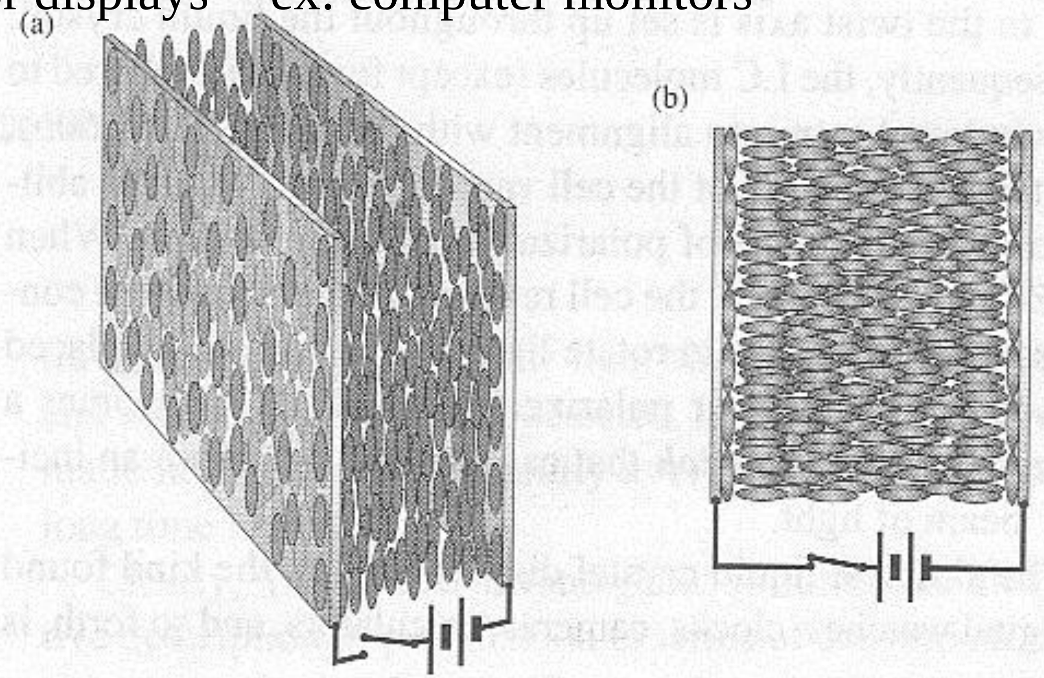
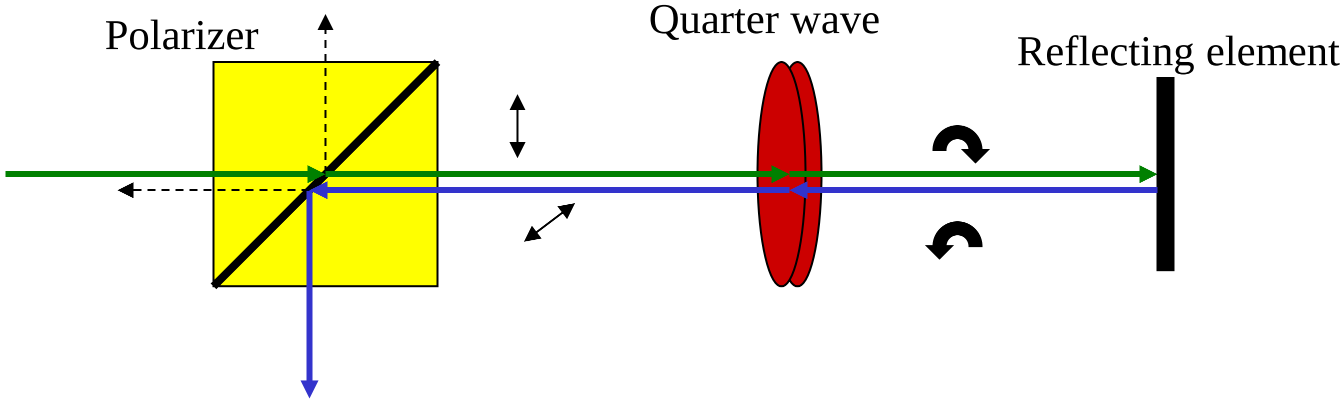


Figure 8.59 (*a*) A nematic liquid crystal between two transparent electrodes. The long molecules align parallel to a set of microgrooves on the inside faces of the two electrodes. (*b*) When a voltage is applied, the molecules rotate into alignment with the field.

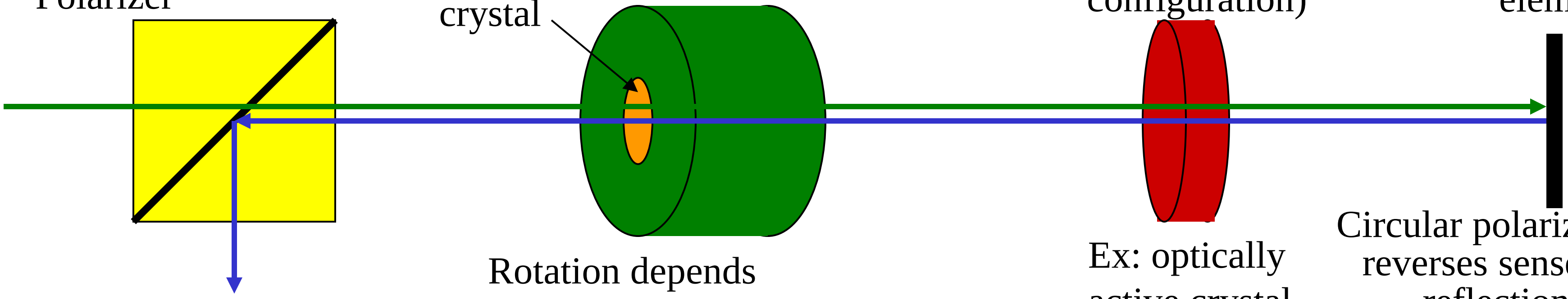
Isolators -- 1

- Polarizer and quarter waveplate
- Double pass through quarter wave plate
  - same as half wave plate
  - rotate polarization by up to 45°
- Polarizer blocks reflected light



Isolators -- 2

- Faraday effect non-reciprocal
  - Opposite for different propagation directions
- Put passive polarization rotator and Faraday rotator in series
  - One direction -- no effect
  - Opposite direction -- rotate polarization 90°
- Polarizer blocks reflections
- Used for fiber optics, laser diodes
  - performance good -- 10 - 30 dB



Mathematical description of polarization

- Stokes vectors
- Elements give:
  - 1/2 total intensity
  - horizontal linear
  - +45° linear
  - right circular

$$\frac{2}{I_0}\begin{pmatrix} I_0/2 \\ I_0/2 - I_H \\ I_0/2 - I_{+45} \\ I_0/2 - I_R \end{pmatrix}$$

Unpolarized state only I_0 is non-zero

- Jones vectors
- Elements give
  - E-field x-component
  - E-field y-component

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

- Only applicable to polarized light

Stokes and Jones vectors

- Special cases of pure polarization

TABLE 8.5 Stokes and Jones Vectors for Some Polarization States

State of polarization	Stokes vectors	Jones vectors
Horizontal $\mathscr{P}$-state	$\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\end{bmatrix}$
Vertical $\mathscr{P}$-state	$\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$	$\begin{bmatrix}0\\1\end{bmatrix}$
$\mathscr{P}$-state at +45°	$\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$
$\mathscr{P}$-state at −45°	$\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$
$\mathscr{R}$-state	$\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$
$\mathscr{L}$-state	$\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$

Action of optical elements

- matrix

Example -- vertical polarizer

- input H state -- zero output
- input V state -- no effect

$$\frac{1}{2}\begin{pmatrix}1 & -1 & 0 & 0\\ -1 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

Jones and Mueller matrices

- Stokes parameters used in:
  - Target ID -- spectral-polarmetric
  - Quantum computing
    - V,H and +45,-45 entangled
    - can only measure in one basis
    - measurement destroys info in other basis

TABLE 8.6 Jones and Mueller matrices.

Linear optical element	Jones matrix	Mueller matrix
Horizontal linear polarizer ↔	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Vertical linear polarizer ↕	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Linear polarizer at +45° ↗	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Linear polarizer at −45° ↖	$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Quarter-wave plate, fast axis vertical	$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Quarter-wave plate, fast axis horizontal	$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$
Homogeneous circular polarizer right ↻	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$
Homogeneous circular polarizer left ↺	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$